A UNIFORM APPROACH TO CONCEPTUAL DESIGN OF AXIAL TURBINE / COMPRESSOR FLOW PATH

Leonid Moroz, Yuri Govorushchenko, Petr Pagur
SoftInWay, Inc.
35 Corporate Dr., Burlington, MA
Tel: (781)685-4942
lm@softinway.com
www.softinway.com

ABSTRACT

A uniform approach to solution on a conceptual level of the axial turbines and compressors design problems was successfully used at elaboration of the turbomachines flowpath integrated CAD system AxSTREAM.

Its essence is in the uniform storing and design data presentation, design problems nomenclature (including optimization analysis) and above all in the developed algorithms methodic similarity of the geometric, strength, aerodynamic analysis and solvers with reference to the axial turbomachines’ elements. The subject problems list associated with flowpaths aerodynamic consists of: multistage flowpath preliminary design, direct and inverse 1D analysis, 2D analysis, plane sections profiling and its flow calculation, airfoil design. If it was possible, authors tried to avoid the variety of mentioned approaches to solution in many respects induced by the historically existing features of turbine and compressor schools development rather than objective causes. In a number of procedures, approaches peculiar to compressor engineering were applied to a turbine calculations that was favorable for a turbine subsystem enhancement.

In the view of the end user the uniform approach facilitates mastering of the software advanced features; in the view of developers - further expansion of the system functionality, software flexibility, integrity, and support ensuring.

INTRODUCTION

In recent years methods of turbomachines flowpaths calculations were being greatly improved owing to widespread introducing of viscous flow 3D calculations (Denton, 1994 and ERCOFTAC, 1997). Nevertheless at present it is obvious for practical specialists that 3D methods utilization does not exclude the 1D and 2D calculations necessity especially at early stages of the design. The simplified methods thanks to its fast response, reliability, and sufficient admission of obtained results sometimes commensurable with 3D viscous calculations (Moroz et al. 2005) can serve as a good base for multivariate, optimization, and multidisciplinary calculations (Moroz L., Govorushchenko, 2004).

There gained a rich variety of software for design and simplified axial turbines and compressors flowpath aerodynamic calculations in the turbomachinery field. However in view of the number of certain reasons these introducing to a design organization practice are highly difficult. The problem solution is in particularized software packages for turbomachines flowpaths conceptual design creation. It is appropriate to include the preliminary design, meanline and quasi axisymmetric flowpath verification calculations, automated profiling, and cascade flow calculation, at least stress and vibration characteristics approximate estimation procedures in the nomenclature of the special problems are solved in the context of similar products. The availability of centralized information storage, uniform interface for control of the design process, inputting and mapping data, optimization subsystem, user modules connection possibilities for losses estimation, working medium properties, etc. provides large conveniences for software users. It is extremely complicate to meet all these claims gathering the package from dissimilar, wrote at different times by different authors software components (of special note are the applied modules) that difficult to integrate, attend, improve, and adapt to specific customer requirements.

It should be pointed out that simplified methods of the flowpath aerodynamic calculation in distinction from the 3D problems are characterized by the wide variety and particular realization orientation. That is in many respects explained by the difference between compressor and turbine community, steam and gas turbines, axial and radial machines engineers, etc.

AxSTREAM software initially based on the long experience of the axial turbines design (steam turbines in general) (Boiko, Govorushchenko, 1989) including set of modules conceived and realized as a uniform system satisfying listed above requirements (Moroz et al., 2004). A “turbine” procedures functionality building, in
particular, cooled gas turbines calculation did not call any principle difficulties. Axial compressors design and analysis problems introduction was decided to realize on the methodical base proposed for axial turbines with the best use of already existing invariant subsystems scopes.

In the first section of the article the architecture of axial turbomachines conceptual design integrated system is briefly described so as to pick out elements the most subjected to changes through the new functionality connection. Then an attempt of outlining axial turbomachine flowpath 1D aerodynamic calculation theory basis that describe process in turbine as well as in compressor is made. Next as an example of a uniform approach turbine and compressor multistage flowpaths preliminary design procedures are described that have received little attention in literature in contrast to verification calculations. In concluding section axial turbine and compressor flowpath design examples are given for software scopes demonstration.

For convenience of the main material perception there are some computations in the appendix.

NOMENCLATURE

Variables

- F area
- G mass flow rate
- H rothalpy
- H_i load factor
- L_a specific work
- N capacity
- R reaction
- c absolute velocity
- d diameter
- i specific enthalpy
- l blade length
- p pressure
- s specific entropy
- u tangential velocity
- w relative velocity
- α flow angle in absolute frame
- β flow angle in relative frame
- δ flow deviation angle
- ε accuracy
- ζ relative loss
- η efficiency
- π pressure ratio
- ρ density
- φ, ψ stator/rotor velocity coefficients
- ω rotation speed
- Θ total pressure loss coefficient

Functions

- I (p, s) specific enthalpy
- P (i, s) pressure
- S (p, i) specific entropy
- i* cascade design incidence
- δ* flow deviation angle
- η (G, p_0, i_0, p_2n...) turbine estimated efficiency

Subscripts

- 0 at the stage inlet, at the beginning of the process
- 1 in section 1
- 2 in section 2
- 3 in section 3
- N related to the nozzle
- R related to working wheel
- I internal (efficiency)
- as as assigned
- h hub
- t in the isentropic process
- u peripheral
- w in the relative frame
- z axial

Superscripts

* stagnation parameters,
cascade design incidence

SYSTEM ARCHITECTURE

Architecture of the modern system of turbomachinery flowpath conceptual design has to match a number of requirements:

- involve a set of design modules necessary for design procedures performing in 1D and 2D formulation with the blade crown 3D geometric models availability for final refinement by means of 3D aerodynamic and stress calculations;
- make it possible to automate multivariate and optimization calculations using putted in system models;
- ensure an interactive design scenario realizing adaptability with the opportunity of return at the early stages, versions support, project integrity, etc;
- to give user convenient mechanisms for input, change, reflection data, accounting forming and data export into the other systems;
- ensure expansibility, scalability, and maintainability.

Conceptual design system subsystems are divided into application (meant for subject problems for design object of the specified type) and invariant (independent of the design object and the problem being solved). As may be seen on the Fig.1 a major subsystems part is related to invariant and does not require essential alteration at addition in the system new applied models and methods. Components gain an access to projects data and reference book through the associated mechanisms with data base only and never exchange data with each other. That's why there is no need to rewrite already existed software components at introduction into the system new models or procedures and it is requires extending data nomenclature.

Inclusion in the system axial compressor calculation and design has required new procedures development of the multistage flowpath preliminary design and inverse stage calculation, flowpath meanline verification.
calculation and stage by stage axisymmetric calculation. In the plane cascades profiling subsystem new method of the profile design by mean line and thickness distribution has added which in some cases turned out a convenient tool for turbine cascade design. Flow calculations along profile outline, boundary layer, and profile losses procedures turned out to be suitable for compressors cascades practically without any changes.

Further we shall take a close look at the axial turbomachines multistage flowpath 1D analysis and synthesis problems because of article size do not allows 2D calculations features consideration that are however in many respects coincide with 1D one.

**AXIAL TURBINES AND COMPRESSORS 1D ANALYSIS**

In literature even elementary theory of axial turbomachines is outlined separately. In the compressor subsystems elaboration process "from the ground" clearly a will arise to use mechanism applied earlier in axial turbine design and analysis down to the limit.

One dimensional steady equilibrium adiabatic flow in the flow path in a reference frame rotating with the speed \( \omega \) is described by a system of equations (Boiko, Govorushchenko, 1989):

1. **energy equation**
   \[ H = i + \frac{w^2 - u^2}{2}; \]  
   (1)

2. **continuity equation**
   \[ G = \rho F \omega w_z; \]  
   (2)

3. **process equation**
   - based on velocity loss coefficient
     \[ s_0 = S(p, \frac{1}{\psi} (i - (1 - \psi^2)i_w^*)); \]  
     (3a)
   - based on total pressure loss coefficient
     \[ s_0 = S\left(\frac{p_w^* - \omega p_0}{1 - \omega}, i_w^* - \frac{u^2}{2} + \frac{u_0^2}{2}\right); \]  
     (3b)

4. **state equations**
   \[ \rho = \rho(p, i); \quad s = S(p, i); \quad \rho = P(i, s); \quad i = I(p, s); \]  
   (4)

To solve this system of equations in direct formulation, the parameters required are as follow:
- stage entry total pressure and enthalpy \( p_0^*, i_0^* \);
- inlet flow angle;
- pressure downstream of stage \( p_{2a} \) for turbine or \( p_{3a} \) for compressor;
- rotational speed \( \omega \);
- mean diameters and blade lengths of cross-sections;
- cascade gauging (for turbine) or geometry (for compressor) exit angles.
- data for additional energy loss computation: seal types and dimensions, radial and axial gaps dimensions, etc.

Denote the nozzle output section by 1 in turbine stage and in compressor by 3.

Then regardless of machine type denote inlet station by 1, outlet station by 2.

Transform the equation of continuity downstream the vane into equation with unknown \( c_1 \) (\( c_3 \)) and \( G \) for turbine stage:

\[ G = \rho (p_1, i_1) c_1 \sin \alpha F_1; \]  
(5a)

for compressor stage:

\[ G = \rho (p_3, i_3) c_3 \sin \alpha F_3; \]  
(5b)

Similarly, for the equation of continuity downstream of working wheel we obtain:

\[ G = \rho (p_2, i_2) w_2 \sin \beta F_2, \]  
(6)

where

\[ H = i_1 + \frac{c_1^2}{2} - u_1 c_{1w}; \]
\[ p_1 = P \left( \frac{i_0^* - c_1^2}{2 \omega^2}, s_0^* \right); \]
\[ i_1 = i_0^* - \frac{c_1^2}{2}; \quad s_1 = S(p_1, i_1); \]
\[ i_2 = H + \frac{u_2^2}{2} - \frac{w_2^2}{2}; \]

for turbine stage:

\[ p_2 = P \left( H + \frac{u_2^2}{2} - \frac{w_2^2}{2 \psi^2}, s_1 \right); \]  
(7a)

for compressor stage:

\[ p_2 = P \left( i_2, s_2 \right); \]
\[ i_3 = i_2^* - \frac{c_3^2}{2}; \quad s_3 = S(p_2^* - \omega \Delta \left(p_2^* - p_2 \right), i_3^*); \]
\[ p_3 = P \left( i_3, s_3 \right); \]

This equation includes unknown quantities \( G_i, c_1, w_2 \) (for turbine) or \( G, w_2, c_1 \) (for compressor).

The third equation is as follows:

For turbine stage:

\[ p_2 = p_{2as}; \]  
(7a)

for compressor stage:

\[ p_3 = p_{3as}; \]  
(7b)
If velocity coefficients $\varphi$ and $\psi$ or total pressure loss coefficients $\theta_X$ and $\theta_Y$ are known, then there are three equations to determine unknown $G$, $c_1$, $w_2$ or $G$, $w_2$, $c_3$. In a general form, these are as follows:

For turbine stage (5a, 6, 7a):

$$g_1(G, c_1) = 0;$$
$$g_2(G, c_1, w_2) = 0;$$
$$g_0(G, c_1, w_2) = 0. \quad (8a)$$

For compressor stage (6, 5b, 7b):

$$g_2^c(G, w_2) = 0;$$
$$g_1^c(G, w_2, c_1) = 0;$$
$$g_0^c(G, w_2, c_3) = 0. \quad (8b)$$

The system is solved numerically by minimizing the sum of disparity squares $g_1^2 + g_2^2 + g_0^2$ or $g_2^2 + g_3^2 + g_0^2$ by a method of conjugated gradients. With regard to the methods used, the multistage flow path computations are the same as for the single stage analysis.

Equations (1)-(4) with boundary conditions describe 1D flow through the turbine as well as compressor. Physical distinction in the processes is that the specific work $L_u = u_c c_1 - u_2 c_{2u}$ turns out (in view of defined by cascade geometry angle of the flow input or output) either positive (in turbine) or negative (in compressor) and accordingly the flow expansion (up to pressure $p_{2a}$) or flow compression (up to pressure $p_{3a}$) takes place.

In the equations (5), (6) cascade outlet flow angles in general case are complicated function of the flowpath geometric parameters (Aungier, 2003). If for turbine
flow angles can be taken equaled to cascade gauging angles with reasonable reliability (and should be recalculated at cascade outlet flow supersonic velocities) so for compressor stages calculations the correct determination of the flow deviation angles from geometric ones is critical.

Velocity coefficients (Craig and Cox, 1970) and total pressure loss coefficients (Aungier, 2003) depending on cascade geometric and mode parameters can be either adjusted in external with relation to solution equations (8a) or (8b) iterations or calculated directly when solving. We know from experience that at compressors flowpath calculations it is more advisable to apply the direct calculation of loss coefficients; it takes place when dependences for them are assigned analytically (not tabulated data) only.

In the current version the turbomachinery flowpaths 1D calculation is still realized by the different software modules but it is clear that in the future they can be combined in a single solver.

**PRELIMINARY DESIGN PROCEDURES**

**Preliminary design algorithm**

Preliminary design is a rapid turbomachine (multistage) flowpath design procedure at minimum information usually includes inlet gas-dynamic parameters, backpressure (capacity), mass flow rate, rotational speed, dimensions, design restrictions. The result of preliminary design is a construction satisfying assigned parameters, restrictions, and ensuring the maximum of the chosen performance criterion, for example efficiency.

Axial turbines preliminary design (Govorushchenko et al., 1991) supposing stage heat calculation inverse 1D problem utilization includes flowpath synthesis procedure, automated design, calculation and algorithm of optimization with a quasirandom search use. This method was applied also to the multistage axial compressor preliminary design. The best solution search is realized in accordance with a scheme shown on the Fig.2.

In a general way method assumes process development with flowpath efficiency approximate evaluation utilization and number of stages calculation by the assigned average loading level. Next, meridian dimensions and cascade angles can be determined on the assumption about the fulfillment of some condition on the stage uniformity (congruence). A part of the geometric parameters like chords, gaps, and others are assigned according to the set of rules making an object of the computer-aided design. Flowpath synthesis is completed with “detailed” inverse stage by stage aerodynamic calculation (see APPENDIX).

**Formula on a pseudo code are adduced by means of the turbines and compressors flowpaths synthesis stages can be realized (with variants).**

**Axial Turbine Synthesis Sequence**

Given: \( p_0 \), \( i_0 \), \( p_{2n} \), \( G \), \( \omega \), \( d_1 \), \( l_1 \), \( a_0 = 90 \), \( R_h \);

**Turbine**

\[
\begin{align*}
    s_0 &= S \left( p_{0s}^*, \ i_{0s} \right); \quad p_0 = \rho \left( p_{0s}^*, \ i_{0s} \right); \\
    H &= i_0 - I \left( p_{2n}, \ s_0 \right); \\
    \eta &= \eta \left( G, \ p_0, \ i_0, \ p_{2n}, \ldots \right); \\
    i_{2n} &= i_0 - H \eta; \\
    s_{2n} &= S \left( p_{2n}, \ i_{2n} \right);
\end{align*}
\]

**1-st stage**

\[
\begin{align*}
    d_{1h} &= d_1 - l_1; \\
    D/l &= d_1/l_1; \\
    R &= 1 - (1 - R_h) \left( 1 - 1 /(D/l) \right)^{1.8}; \\
    \text{guess} p_1 &= p_0;
\end{align*}
\]

**Iterative more precise density definition:**

\[
\begin{align*}
    c_{1u} &= 2 \ u_1 \left( 1 - R \right); \\
    c_{1s} &= G / (\pi \ d_1 \ l_1 \ \rho_1); \\
    \alpha_1 &= \arctg \left( c_{1s} / c_{1u} \right); \\
    c_1 &= c_{1u} / \cos (\alpha_1); \\
    \nu_{th} &= u_{1h} / C_0; \\
    n_s &= \text{floor} \left( 8 \ H / \left( (\omega \ d_{1h} / \nu_{th})^2 - c_{1s}^2 / 2 \right) + 0.5 \right); \\
    \Delta i &= (i_0 - c_{1s}^2 / 2 - \ i_{2n}) \ / \ ns; \\
    \Delta s &= (i_{2n} - s_0) / \ ns; \\
    \Delta H &= (\Delta i + c_{1s}^2 / 2) / \ \eta; \\
    p_2 &= P_i \left( i_0^* - c_{1s}^2 / 2 - \Delta i, \ s_0 + \Delta s \right); \\
    p_1 &= P_i \left( i_0^* - (1 - R) \ \Delta H, \ s_0 \right); \\
    i_1 &= I \left( p_1, \ s_0 + (1 - R) \ \Delta s \right);
\end{align*}
\]
\[ \rho_1 = \rho (p_1, i_1); \]

until \( \rho_1 \) converged

**Other Stages** \((1 < j < n)\)

\[ c_{1z,j} = r_{1z} (1 + (K_j - 1) \xi / (n_j - 1)); \]

\[ l_{0,j} = l_{0,1} - c_{1z,j} / 2; \]

\[ s_{0,j} = s_0 + j \Delta s; \]

\[ \Delta H = \Delta i / \eta + c_{1z,j} / 2; \]

guess \( R_j = 0; \)

Iterative more precise density definition:

\[ p_{2z,j} = P (l_{0,j} - c_{1z,j} / 2 - (j + 1) \Delta i, s_0 + (j + 1) \Delta s); \]

\[ p_{1,j} = P (l_{0,j} - (1 - R_j) \Delta H, s_{0,j}); \]

\[ l_{1,j} = I (p_{1,j}, s_{0,j} + (1 - R_j) \Delta s); \]

\[ p_{1,j} = P (p_{1,j}, s_{0,j}); \]

\[ l_{1,j} = 0.5 (D_h + (D_h^2 + 4 G (\pi p_{1,j}^2 c_{1z,j}))^{1/2}); \]

\[ R_j = 1 - (1 - R_j) (1 - l_j / D_h)^{1/2}; \]

until \( p_{1,j} \) converged

Design

“Detailed” turbine inverse stage by stage calculation

**Axial Compressor Synthesis Sequence**

Given: \( p_0^*, i_0^*, p_{2n}, G, \omega, d_1, l_1, a_{1,1} \);

\[ s_0 = S (p_0^*, i_0^*); \]

guess \( p_{1,1}, c_{1,1} \)

Iterative more precise density definition at inflow:

\[ c_{1z,1} = c_{1,1} \sin (\alpha_{1,1}); \]

\[ l_{0,1} = l_{0,1} - c_{1z,1} / 2; \]

\[ p_{0,1} = P (i_{0,1}, s_0); \]

until \( p_{1,1} \) converged

\[ p_{2,1} = p_{0,1}, \pi; \]

\[ l_{1,1} = I (p_{2,1}, s_0); \]

\[ c_{1z,1} = c_{1z,1} / \sin (\alpha_{1,1}); \]

\[ l_{1,1} = l_{1,1} + c_{1z,1} / 2; \]

\[ \eta = \eta (\ldots); \]

\[ i_{1,1} = i_{0,1}^* + (i_{1z,1} - i_{0,1}^*) / \eta; \]

\[ c_{1z,1} = c_{1z,1} - c_{1z,1} / 2; \]

\[ \rho_{1,1} = \rho (p_{2,1}, i_{1,1}); \]

\[ l_{1,1} = G (\omega, p_{1,1}, d_1, l_1); \]

\[ u_1 = \omega d_1 / 2; \]

\[ n_s = \text{cel} (i_{1,1} - i_0) / (u_1^2 H_1); \]

\[ c_{1z,1} = (c_{1z,1} + c_{2z,1}) / 2; \]

\[ n_0 = \max (1, n_s / 2); \]

\[ L_{0,0} = (i_1 - i_0) / n_0; \]

Iterative more precise loading definition:

\[ \eta_0 = \eta; \]

for all stages:

\[ c_z = c_0 (1 + (K_z - 1) \xi / (n_z - 1)); \]

\[ L_{0,0} = L_{0,0} (1 + (K_0 - 1) \xi / (n_0 - 1)); \]

\[ c_{1z,0} = c_0 (1 + (K_0 - 1) \xi / (n_0 - 1)); \]

\[ d_{1,j} = d_{2,j} = d_{1,j}; \]

\[ a_{1,1} = a_{1,1} \ldots \text{congruous stages}; \]

\[ H_{1,j} = L_d / u_{1,j}; \]

“Design”

“Detailed” inverse stage by stage calculation

\[ \pi = p_{2,1}/p_{1,1}; \]

\[ \eta = (i_{1z,1} - i_0) / (i_{1z,1} - i_0); \]

\[ L_{0,0} = L_{0,0} \pi_0 / \pi; \]

While \(|\eta_0 - \eta| > \varepsilon \) or \(|1 - \pi_0 / \pi| > \varepsilon \).

The formal algorithm extensibility gives engineer an opportunity of the turbine and compressor type flowpaths synthesis problems of wide range of choice.

**EXAMPLES OF A TURBINE AND COMPRESSOR PRELIMINARY DESIGN**

To demonstrate a software realization potentiality of the described axial turbines and compressors multistage flowpaths analysis and synthesis methods let’s consider some examples.

**Axial turbine**

Gas turbine flowpath with parameters indicated below has designed with the constant hub and tip diameters.

<table>
<thead>
<tr>
<th>Gas turbine design parameters</th>
<th>products of combustion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working medium</td>
<td>530000 Pa</td>
</tr>
<tr>
<td>Inlet pressure</td>
<td>1190000 J/kg</td>
</tr>
<tr>
<td>Inlet Enthalpy</td>
<td>90 kg/s</td>
</tr>
<tr>
<td>Mass flow rate</td>
<td>5000 rps</td>
</tr>
<tr>
<td>Rotation speed</td>
<td>180000 Pa</td>
</tr>
<tr>
<td>Outlet pressure</td>
<td>1...4</td>
</tr>
<tr>
<td>Restrictions</td>
<td>no restrictions</td>
</tr>
<tr>
<td>Number of stages</td>
<td>0.05..0.15 m</td>
</tr>
<tr>
<td>Tip diameter</td>
<td>0..0.2</td>
</tr>
<tr>
<td>Hub reactivity</td>
<td>700</td>
</tr>
<tr>
<td>Search points</td>
<td></td>
</tr>
</tbody>
</table>

The results of considered variants are given in tab. 1, 2. Because of the stage uniformity there are given velocity triangles only for the first stage.

**Conventional hub diameter 0.6...0.8 m**

<table>
<thead>
<tr>
<th></th>
<th>( \alpha_1 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \alpha_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi )</td>
<td>30.658004</td>
<td>70.857800</td>
<td>34.033186</td>
<td>80.022138</td>
</tr>
<tr>
<td>( \pi )</td>
<td>0.528328</td>
<td>0.370871</td>
<td>0.769378</td>
<td></td>
</tr>
</tbody>
</table>

Turbine integral characteristics

by inverse problem:

\[ G = 90 \text{ kft} / \text{c}, N = 24.0879 \text{ MW}, \eta_t = 0.8912 \]

by direct problem:

\[ G = 89.989, N = 24.35 \text{ MW}, \eta_t = 0.8963 \]
Fig. 3. Turbine flowpath and velocity triangles of the first stage (constant hub diameter).  

Constant tip diameter  1.0...1.6  m  

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>deg</td>
</tr>
<tr>
<td>B₁</td>
<td>deg</td>
</tr>
<tr>
<td>B₂</td>
<td>deg</td>
</tr>
<tr>
<td>A₂</td>
<td>deg</td>
</tr>
<tr>
<td>u/C₀</td>
<td>-</td>
</tr>
<tr>
<td>R</td>
<td>-</td>
</tr>
<tr>
<td>H</td>
<td>-</td>
</tr>
</tbody>
</table>

This case integral characteristics by inverse problem:  
\[ G_0 = 90 \text{ kg/s}, \quad N = 24.3742 \text{ MW}, \quad \eta = 0.901792 \]

by direct problem:  
\[ G_0 = 89.982 \text{ kg/s}, \quad N = 24.34 \text{ MW}, \quad \eta = 0.9007 \]

Fast response of the turbine preliminary design accomplishment should be noted as it may be restricted by moderate (of the hundredth order) number of a random search points.

Axial compressor  
Axial compressor flowpath with parameters indicated below has designed with the constant hub and tip diameters.

Axial compressor  
Axial compressor flowpath with parameters indicated below has designed with the constant hub and tip diameters.

Table 2.  

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>18.688596</td>
</tr>
<tr>
<td>B₁</td>
<td>73.398619</td>
</tr>
<tr>
<td>B₂</td>
<td>24.766969</td>
</tr>
<tr>
<td>A₂</td>
<td>111.896133</td>
</tr>
<tr>
<td>u/C₀</td>
<td>0.675703</td>
</tr>
<tr>
<td>R</td>
<td>0.340574</td>
</tr>
<tr>
<td>H</td>
<td>0.855622</td>
</tr>
</tbody>
</table>

Table 3.  

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>β₁</td>
<td>43.189624</td>
</tr>
<tr>
<td>α₂</td>
<td>124.338411</td>
</tr>
<tr>
<td>β₂</td>
<td>73.279359</td>
</tr>
<tr>
<td>α₃</td>
<td>90.000000</td>
</tr>
<tr>
<td>Hᵣ</td>
<td>0.642297</td>
</tr>
<tr>
<td>R</td>
<td>0.594526</td>
</tr>
<tr>
<td>η</td>
<td>0.853064</td>
</tr>
</tbody>
</table>
Fig. 5. Compressor flowpath and velocity triangles of the first stage (constant hub diameter)

Compressor integral characteristics
by inverse problem: $G_0 = 15$ kg/s, $\eta_i = 0.858088$
by direct problem: $G_0 = 14.961$ kg/s, $\eta_i = 0.8690$

**Constant tip diameter 0.3...0.5m**

<p>| | | |</p>
<table>
<thead>
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<tr>
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</tr>
<tr>
<td>B2</td>
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<td>72.502638</td>
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<td>A3</td>
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<tr>
<td>H_1</td>
<td>-</td>
<td>0.697492</td>
</tr>
<tr>
<td>R</td>
<td>-</td>
<td>0.651254</td>
</tr>
<tr>
<td>$\eta$</td>
<td>-</td>
<td>0.927679</td>
</tr>
</tbody>
</table>

Compressor integral characteristics
by inverse problem: $G_0 = 15$ kg/s, $\eta_i = 0.920019$
by direct problem: $G_0 = 14.438$ kg/s, $\eta_i = 0.9157$

Number of the random search points at compressor preliminary design should be taken considerably more in contrast to turbine (of the thousandth order). It is caused by “rarity” of the permissible solutions in the compressor design parameters space.
CONCLUSION

The extension of the axial turbines flowpaths analyses and synthesis engineering principles to the axial compressors in the context of its utilization in the turbomachines integrated CAD system. An experience suggests about possibility of a more detailed theoretical method elaboration of the turbines and compressors design. It made possible to generalize previously designed software and with the various invariant subsystems utilization to extend turbine subsystem functionality to compressor components rapidly and surely. In the short term it is assumed to extend software potentiality to the radial turbomachines and mixed type flowpaths.

REFERENCES


Govorushchenko Y., Romanov Г.Л., Скибина Е.Э.
“Ав том а т и з и р о в а н ное
п р е д в а р и т е л ь ное
п ро е к т и р о в а н и е
проточной части
многоступенчатых
паровых турбин”.


APPENDIX

Turbine stage inverse calculation

Given: \( p_0, i_0, p_2, G, \Omega, R, d_1, d_2, l_1(c_{1z}), l_2(c_{2z}). \)

Stage inflow (section 0)

\[
T_0 = T(p_0, i_0); \\
s_0 = S(p_0, i_0); \\
p_0 = \rho(p_0, i_0); \\
i_{2w} = I(p_2, s_0); \\
H = i_0 - i_{2w} = c_0^2 / 2;
\]

Nozzle outflow (section 1)

\[
c_{1i}^2 = i_0^2 - c_{1i}^2 = H (1 - R); \\
c_1 = a_{10} s_1; \\
\Delta h_N = (1 - R) c_1^2 / 2; \\
i_1 = i_{1i} + \Delta h_N; \\
p_1 = \rho(p_1, i_1); \\
s_1 = S(p_1, i_1); \\
c_{1z} = G / (\pi d_1 i_1 p_1); \\
l_1 = G / (\pi d_1 p_1 c_{1z}); \\
c_{1w} = c_1 - c_{1z}; \\
\alpha_{1w} = arc \theta (c_{1w} / c_0);
\]

Rotor outflow (section 2)

\[
i_{2i} = I(p_2, s_1); \\
w_{2i}^2 = i_1 + (w_{1i}^2 - i_1^2) / 2 - (i_2 - u_2)^2 / 2; \\
w_2 = \psi w_{2i}; \\
\Delta h_R = (1 - R) w_2^2 / 2; \\
i_2 = i_{2i} + \Delta h_R; \\
c_2 = S(p_2, i_2); \\
p_2 = \rho(p_2, i_2); \\
c_{2z} = G / (\pi d_2 i_1 p_2); \\
l_2 = G / (\pi d_2 p_2 c_{2z}); \\
w_{2w} = w_{2i}^2 - c_{2w}; \\
\beta_{2w} = 180 - arc \theta (c_{2w} / w_{2w}); \\
\varphi = \varphi (...) ; \\
\psi = \psi (...) ;
\]

Stage

\[
L_0 = u_1 c_{1w} - u_2 c_{2w}; \\
\eta_0 = L_0 / H; \\
\eta_1 = \eta_0 - \Sigma \Delta h_i / H; \\
N_c = H G \eta_0; \\
\]

Compressor stage inverse calculation

Given: \( p_0, i_0, H, G, \omega, \alpha_1, a_1, d_1, d_2, d_3, c_{2z}, c_{3z}. \)

Stage inflow (section 1)

\[
c_{1u} = c_{1z} / \Omega (a_1); \\
c_{1u}^2 = c_{1w}^2 + c_{1z}^2; \\
w_{1u} = c_{1u} - u_1; \\
\beta_1 = 180 - arc \theta (c_{1w} / w_{1w}); \\
s_1 = S(p_1, i_1); \\
i_1 = i_{1i} - c_{1i}^2 / 2; \\
p_1 = \rho(p_1, i_1); \\
l_1 = G / (\pi d_1 p_1 c_{1z}); \\
\]

Rotor outflow (section 2)

\[
L_2 = H u_1^2; \\
i_2 = i_{2w} + L_u; \\
c_{2w} = (L_u + i_1 c_{1w}) / u_2; \\
c_{2w}^2 = c_{2w}^2 + c_{2w}^2; \\
\alpha_2 = arc \theta (c_{2w} / c_{2w}); \\
w_{2w} = c_{2u} - u_2; \\
w_{2w}^2 = w_{2w}^2 + c_{2w}; \\
\beta_2 = 180 - arc \theta (c_{2w} / w_{2w}); \\
i_2 = i_{2w}^2 - c_{2w}^2 / 2; \\
i_{2w} = i_2 + w_{2w}^2 / 2; \\
\beta_{2w} = \beta_1 + i_{2w}^2; \\
\beta_{2w} = \beta_2 + \delta_{2w}; \\
p_{2w} = \rho(p_{2w}, i_{2w}) - (p_{1w} - p_1) (1 - \omega_R); \\
s_2 = S(p_{2w}, i_{2w}); \\
p_2 = \rho(p_2, i_2); \\
l_2 = G / (\pi d_2 p_2 c_{2z}); \\
\]

Nozzle outflow (section 3)

\[
i_3 = i_2; \\
c_3 = c_{1z} / c_{1z}; \\
i_{3x} = i_{3x} - c_{1z}^2 / 2; \\
R = (i_2 - i_{3x}) / (i_1 - i_{3x}); \\
c_{3z}^2 = c_{3z}^2 - c_{3z}; \\
w_{3z} = c_{3z} - u_3; \\
w_3 = w_{3w}^2 + c_{3z}; \\
\alpha_3 = arc \theta (c_{3w} / c_{3z}); \\
\beta_3 = 180 - arc \theta (c_{3w} / w_{3w}); \\
\]

Stage

\[
\eta_0 = L_0 / \Omega; \\
\eta_1 = \eta_0 - \Sigma \Delta h_i / \Omega; \\
\]

\[
\]