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Methods and Tools for Multidisciplinary Optimization of Axial Turbine Stages with Relatively Long Blades

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ABSTRACT
An effective methodology for optimal design of axial turbine blades is presented. It has been used for achieving stage maximal efficiency meeting both stress-strain and vibration reliability requirements and taking into account technological limitations.

INTRODUCTION
Problem formulations of the turbine flow path optimization reflect main phases of axial turbines design practice [1] and use increasingly accurate models of elaborated designs. As the industry moves forward, integration of 3D modeling of aerodynamic and strength of material characteristics into optimization process generates continuously increasing interest. Usually, such optimization refers to analysis of separate blade row or isolated stage [2 - 7, 9] and involves a large number of optimization parameters. This leads to enormous computational time and requires significant computational resources. Nevertheless, 3D modeling is an important part of numerical modeling along with conventional 1D and 2D approaches.

This paper describes a process of optimal flow path design that is achieved through the following steps:

- rapid flow path design and optimization using reduced order models and axi-symmetrical solver;
- blade cross-sections profiling according to aerodynamic criteria, blade stacking (3D profiling) with optimized twist/lean;
- generation of parameterized mesh for buckets and parameterized grid for inter-blade passages;
- detailed 3D CFD computations and finite element structural and modal analyses with commercial CFD and FEA tools;
- design optimization using design of experiment (DoE) methods and reduced order models.

Process begins from preliminary flow path design. Tools such as ***AxSTREAM™*** allow significantly reduce the search range for bucket optimal configuration. ***AxSTREAM™*** uses stage and airfoil optimizations that are based on DoE methods in combination with 2D aerodynamic and 1D structural calculations. Computed data can be exported to external tool for mesh and grid generation.

MinuteMesh-Turbo™, a parameterized mesh generator specifically developed for turbomachinery applications can be used as a preprocessor for industry standard CFD and FEA packages. ***MinuteMesh-Turbo™*** generates complete FE models consisting of structured mesh, loads, boundary conditions (BC's) and material properties. Models are optimized for modal, harmonic and structural analyses with FEA solver of choice. FE model could contain one blade, a packet of blades and up to a full bladed disk assembly with all components: airfoils, shroud, tiewires, root, disk, etc. ***MinuteMesh-Turbo™*** also creates a grid of inter-blade flow path for CFD analysis.

AxPLAN™ DoE tool makes possible to decrease a number of time-consuming 3D computations by evaluating the response function sensitivity to varied parameters. It also formulates and solves optimization problems, and acts as pre- and post-processor. Besides this, it is possible to store and re-use reduced order models for quick design of geometrically similar buckets without detailed 3D CFD computations.

Described tools are seamlessly integrated with industry standard 3D CFD and FEA packages and, therefore, can be used by design organizations with minimal changes to established design practices.

NOMENCLATURE
α_1 – nozzle exit angle;
β_2 – blade exit angle;
δ_1 – nozzle lean;
δ_2 – blade lean;
Γ – vector of geometrical parameters;
P – vector of operational parameters;
m_1, m_2 – nozzle and blade twist parameters;
t, T – time;
Q – vector of varied parameters;
Y – vector of response function;
NURBS – non-uniform rational B-spline;

GENERAL BLADE MDO SCHEME

In a bulk of cases, turbine flow path multidisciplinary optimization (MDO) uses single-line approach that assumes certain airfoil parameterization, meshing of parameterized geometry model and separate utilization of the point solvers for aerodynamic, structural and other kinds of analysis [2-7]. Usually, choice of optimization method is reduced to a random search (as a rule, genetic algorithms) or to design of experiment with consequent solving the problem of optimization, Designer has to be able to control a module for building parameterized geometry by assigning the module's parameters, extracting the values of dependent variables from solvers and analyzing the solution in post processor.

Automatic optimization becomes possible if used meshers can, on one hand, interpret a format of parameterized element description and, on the other hand, create the meshes in a format that known commercial solvers can interpret. Clearly, MDO requires development of non-trivial software for data exchange arrangement between optimization module and solvers, and development of the custom meshing tools (Figure 1).

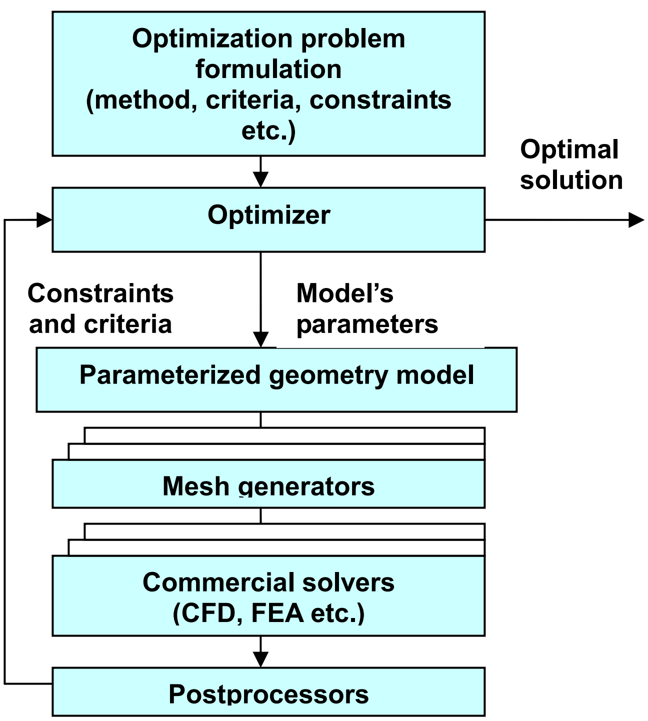


Figure 1: General MDO scheme

In real design practice, MDO principles aren't bounded by 3D modeling and can be applied for reduced aerodynamic and other types of analyses.

In this paper we'll show how the general scheme presented in Figure 1 can be altered if MDO is applied to development of integrated program for axial turbine conceptual design. Software structure consists of the project database, propriety optimizer, embedded modules of turbine elements synthesis and analysis. Operation begins with examination of the multistage flow path and ends with the study of blades with variable cross-section. Solution of the analysis and optimization problems in multidisciplinary formulation including aerodynamic, strength and feasibility studies doesn't present any difficulties at each phase of the process.

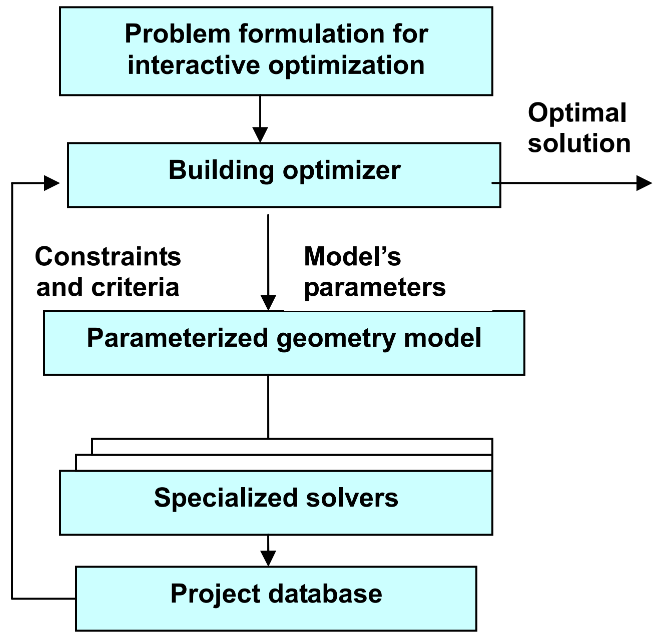


Figure 2: MDO scheme in integrated environment

The scheme is simplified at the account of external solvers, optimizer and mesher dropout (Figure 2). The code can normally combine the interactive formulation of optimization problem and flexible control of the process within the framework of single problem solution.

As a whole, such an approach essentially accelerates all phases of the flow path design that precede full 3D analysis of the buckets, and provides with the design optimized for a majority of required parameters. This allows to minimize a quantity of time-consuming 3D computations in CFD and FEA modeling.

PROBLEMS OF RAPID FLOW PATH DESIGN

Let's examine key phases of axial turbines flow path design with ***AxSTREAM™***. Here the MDO-based approach is applied for all phases of design. The differentiation of ***AxSTREAM's*** optimization methods is in the capability to select an appropriate optimization task supplemented with interactive interference in a search process with simultaneous correction of the latter in order to find the solution most appropriate for designer.

Preliminary flow path design

At this phase of design, a highly efficient multistage flow path that meets a set of technological and design requirements can be developed. The problem is formulated interactively by selecting tasks to be addressed (e.g. steam or gas turbine, impulse or reaction turbine, etc.). First, section's efficiency is estimated approximately with empiric dependencies, then, more thorough stage-by-stage 1D analysis is carried out using reverse problem formulation. Random search with small set of principal parameters (up to 5) is applied for finding the optimal solution. This is convenient since the solution of reverse problems during stage calculation can be achieved rapidly including many certainly inadmissible combinations of chosen parameters. Usually, a number of computational points are estimated at hundreds that takes just a few seconds of computing time.

Stage-by-stage optimization

At the next phase of design, more elaborated optimization of each stage is conducted using reverse 1D aerodynamic and structural computations in combination with random search. Optimization tasks are divided into two groups. First one involves tasks in S1 planes (optimal chord and grid density determination). Second group contains the tasks in S2 planes (stage meridional dimensions, degree of reaction, etc.). Joint computation of the tasks from two groups is possible. Structural and vibration limitations are considered at this phase, that allow to address such aspects of design as, for instance, admissible ratio of blades and nozzles in a crown.

Computation of profiles geometry required for strength evaluation is carried out relatively to a method of profile determination technique: with empiric correlations (doesn't require profile configuration data), by standard profiles selection from profiles database or with the cascades profiled "on-the-fly". For this, the random search method of optimization is used again.

1D off-design analysis and optimization

The multistage flow path designed for indicated operation is analyzed in direct formulation with more accurate 1D solver that makes possible to include influence of through-seals leakage and fluid extractions, and compute axial forces at various values of operational parameters.

The problem can be solved in different formulations with:

- flow rate determination at assigned gas parameters at inlet of the flow path and pressure after it;
- determination of entry pressure that ensures assigned flow rate;
- correction of the nozzle flow exit angles that assures selected flow rate at assigned parameters at the flow path boundaries.

The last formulation is usable for solving different optimization tasks related to determination of the turbine geometrical dimensions and, strictly speaking, can't be considered as direct one, though practically the method of its solution doesn't differ from the first two formulations. We call this last formulation 'the problem of flow path designing calculations' [8].

Solution of the system of equations for 1D flow in axial gaps is based on minimization of sum of residuals squares by conjugated gradients method. It features very robust and fast algorithm for a wide range of operational points even for complicated flow path composed of dozens of stages.

Usage of DoE is completely justified for the design problems of multistage turbines flow path, functioning under variable conditions. The results of this methodology are quadratic dependencies of output parameters on geometrical $\boldsymbol{\Gamma}$ and operational $\boldsymbol{P}$ factors

$$N = F(\Gamma, P).$$

These dependencies are the subjects of independent importance as they help to evaluate turbine sensitivity to variation of geometry and operational conditions. The response functions extracted (i.e. macromodels) can be stored and used repeatedly for solving the different problems including optimization.

By definition [1], multi-operational flow path optimization is a search for the parameters that can provide maximum of mechanical energy in time $\boldsymbol{T}$ or maximum of average capacity in the same period of time at operational conditions varied according to the law $\boldsymbol{P(t)}$:

$$N_{avg} = \frac{1}{T}\int_0^T F\ (\Gamma, P(t))dt\,.$$

Due to integration, operational variables drop out of the last formula and N_{avg} is transformed into quadratic function of geometrical parameters only, which extremum can be easily find by random search.

Axi-symmetrical analysis and optimization

A special method for quasi-2D analysis of the blades twisting laws has been developed. The essence of the method lays in the stage computation with regard to clearances, augmented by an algorithm of ascertainment of streamlines' lean and curvature in computed cross-sections. The method was thoroughly validated by comparison of computational results with experimental data for various types of stages [1] and it was successfully used in comparative trials of baseline and optimized flow paths. Some additional data pertaining to the validation method is presented in Appendix 1.

The equations that describe the flow in axial gaps are presented as a system of ordinary differential equations for velocities and radii of streamlines. Stream function is chosen as an independent variable. The boundary problem is reduced to a system of non-linear equations solved with non-linear programming methods.

The method allows reliably analyze the stages with large angles of the flow path expansion and supersonic flow velocities, and uses the real fluids properties in a wide range of operational parameters variation.

The problem formulations similar to 1D verifying analysis are also possible. Since computing of one of the alternatives takes several dozens of seconds, the random search for blade twist optimization becomes quite a time-consuming procedure since it requires at least hundreds of computations. The method that is far more effective is based on solving the problem by building response functions with the help of DoE method. A small set of the blade twist parameters is used just for several dozens of initial model computations. After this, the optimization with quadratic models is carried out just in a few seconds.

BLADE PROFILING

When optimal laws of the blades twist are defined, the airfoil design begins. The process includes flat sections profiling, parameters coordination along blade's height and blade surface forming. These phases are coupled and should be carried out iteratively.

Blade-to-blade 2D profiling

Experience shows that most effective way of blade profiling is interactive design supplemented with capability to describe profile's properties in generalized and intuitively clear for designer terminology [9]. Along with this, parameterization of the plain cascade for optimal profile shaping becomes possible. We use 7 characteristics for parameterization:

- relative pitch;
- incidence angle;
- geometrical exit angle;
- leading and trailing edge convergence angles;
- leading edge radius;
- leading edge stagger angle.

If the effective exit angle, thickness of the trailing edge, and the chord are given, then there is enough data for profile reconstruction with the method that relies on Bezier curves for description of the suction side (from trailing edge to the throat and from throat to the leading edge), and the pressure side, as well. Basic curves are built on 4 reference points in such a way so that boundary conditions at the points of conjugation are preserved (continuity of 1^{st} and 2^{nd} derivatives), and a minimum of maximum curvature is achieved. In the process of subsonic cascades profiling it is reasonable to compute potential streamlining, boundary layer and profile loss. The parameterization allows to effectively optimize plain cascades using comparatively small set of variables with the help of random search method. Here two criteria of optimality are used:

- geometrical: a minimum of maximum curvature of the suction and pressure sides;
- gasdynamic: a minimum of profile loss.

During optimization all physically unrealizable configurations are discarded, and restrictions on the profile's area are imposed. Finally, a quantity of admissible alternatives is significantly reduced and computing of several hundreds of points takes from a few seconds to several dozens of seconds.

In addition, manual profiling is available by changing the required parameters' values in the data grid.

3D-stacking

Stacking stands for airfoil profiling using plain sections being translated in tangential and axial directions.

Usually, blade stacking is performed along radius on the leading/trailing edges or on the cascade center of mass. But it can be carried on a non-radial line or curve (to arrange a certain lean or sweep, for example) and even virtual arrangement of the cascades along the blade height is possible. Most easily airfoil profiling can be conducted with NURBS using even comparatively low order curves. Convenient and validated by practice steps for airfoil profiling are as follows:

- determination of appropriate method of stacking;
- estimation of basic profile parameters along blade height with empiric correlations;
- automatic cross-sections profiling and airfoil building as first approximation;
- visual inspection of the blade quality (smoothness, streamlining, stress radial distribution) and hands-on correction of cascade parameters variation along height;
- optimized profiling of sections and adjustment of cascade parameters variation along height if needed.

Complete blade automatic profiling requires 3D modeling of viscid flow with small number of airfoil parameters.

PARAMETERIZED MESH GENERATION AND 3D COMPUTATIONS

Generation of finite element models for 3D computations is carried out on the basis of airfoil profiling data. This data defines point-to-point airfoil cross-sections configuration. For the sake of usability, the points are grouped as belonging to leading/trailing edges and pressure/suction sides of the blade. This information is added with parameters that define rim, band, root and hub, Figure 3. Mesh parameters are also included.

According to the airfoil cross-sections, an interpolating NURBS are computed, i.e. the surfaces in parametric space. Internal nodes are created by inwards interpolation between original nodes. The elements obtained at extreme sections of the airfoil are starting points for rim and band meshing. Fillets mesh is generated by transition of the nodes located near line of conjugated surfaces intersection to fillet surface with subsequent smoothing of the mesh near it.

The steps used for disk and root meshing are as follows:

- mesh extrusion along the line combined with mesh transformation;
- generation of the second order macroelements followed by their partition into the first order elements;
- elements deformation according to desired law.

In those regions, where closely located elements of blades are to be separated, a gap is introduced between them that exceeds inter-nodes distance and prevents merging.

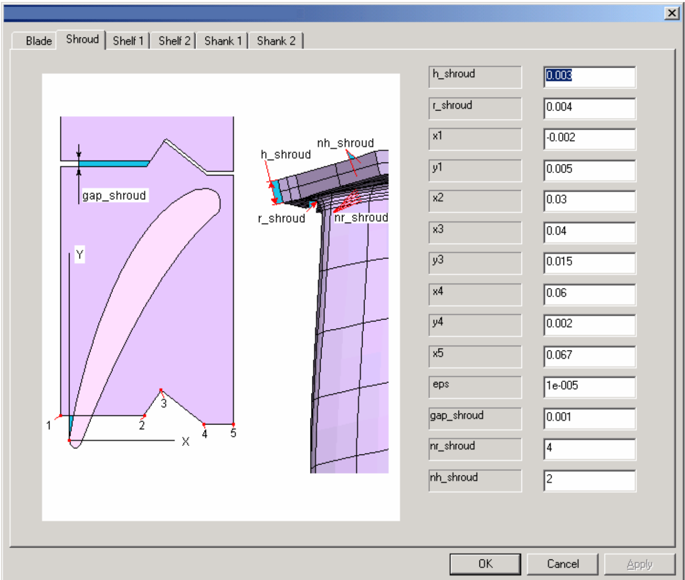


Figure 3. Dialog of initial data input for blade rim

Mesh generated (Figure 4) is stored as an ANSYS commands text file that makes possible computing the airfoil stress-strain state caused by centrifugal inertia forces and steam pressure loads, and also carrying out modal analysis.

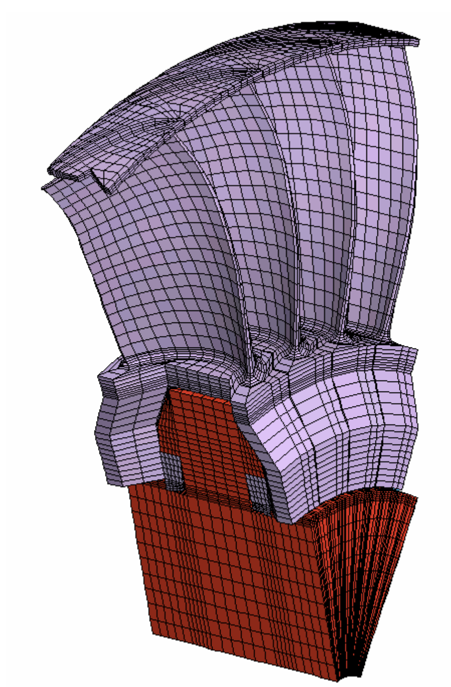


Figure 4. Mesh of finite elements for 4-blade packet and disk sector

At this phase, computations with 3D models are used for verification of the data computed with reduced-order models.

The approach described for structured mesh generation of the blade will be further utilized for development of gasdynamic model of the channel. This will allow implementation of a complete cycle of airfoil optimization on 3D models.

OPTIMIZATION TECHNIQUES

Quasi-random search with Original (OMM) and Formal (FMM) Macromodels

Such features of real design as multi-objectiveness, multi-extremity, high dimensionality, intractability of computations, non-differentiability of target function or constraints, multi-variable region of permissible values, uncertainty of zones, etc. and their combination dictate the choice of optimization methods. In complex cases it is more practical using a designer-computer dialog, rather than a fully automated search for the optimum. Designer's experience and intuition combined with computer capabilities in a number of cases allow to speed up the process noticeably. There are all kinds of random searches, which don't require any assumptions about the target function, need modest computational resources and are a good choice for practice-oriented optimal design in interactive implementation.

Random search based on so called Random Best Succession method (RBS) delivers most uniform results between all evenly distributed successions known [10], and fits well to solving the problems with several dozens of parameters and minimal requirements to smoothness of target function and constraints. This method, likewise the method of direct enumeration, is based on multidimensional regions examination, but in contrast with rectangular grids usage, the RBS search allows significantly reduce the quantity of computational points.

A search for extremum of intractable yet reasonably uniform functions can be expediently done using DoE methodology. An original function is replaced by its quadratic model extracted from results of numerical experiment obtained by computation of the function's values at strategically selected points. If constraints of the optimization are too complicated, they are also substituted by quadratic models and the optimization is performed for approximated target function and constraints with one of the non-linear programming methods. Such approach saves so much computational resources, that there is a great difference in computational times of the original function and its quadratic model. Sometimes, determination of more optimal combination of varied parameters requires development of a new macromodel within narrower vicinity of optimal obtained as the first approximation solution.

Design of numerical experiment

The method relies on a set of "black box" correlations (exponential or polynomials, for example) with reduced (vs. full model) number of inner relationships. Formal macromodels (FMM) can be extracted from results of numerical experiments performed on the original models (OMM).

To create FMM, one can apply DoE methodology and achieve critical reduction in the number of computational points, preserving minimal dimensionality of ***Y*** vector.

The following steps are required for FMM implementation:

1. OMM definition;
2. efficiency criterion selection;
3. selection of those OMM parameters that influence the efficiency criterion and need detailed examination, and forming a vector of varied parameters ***Q***;
4. determination of the region of macromodeling (i.e. ranges of ***Q***-vector components variation);
5. generation a matrix of experiment planning;
6. running numerical experiments and evaluation of components of the response functions making up vector ***Y'***;
7. processing the results of experiment and extraction FMM coefficients.

Steps 1 - 4 couldn't be defined formally; hence, it is necessary to consider specific features of the objects for macromodeling and use accumulated engineering experience. The rest of the steps can be automated with ***AxPLAN™*** DoE tool that allows:

- formulation and planning of experiment;
- using *a priory* known information about correlations between varying parameters and characteristics of the studied object;
- processing of experimental results and extracting the macromodels from found correlations;
- macromodeling of the object characteristics;
- solving multi-criterion optimization problems including every possible constraint;
- visualization of the topology lines in the design space and their interactive examination.

The tool carries out DoE following Box-Benken's approximation plans and Rechtschafner's saturated quadratic plans. Each method has its *pro* and *cons*. For example, 16 numerical experiments are required in order to obtain the quadratic polynomial form of four independent parameters for Rechtschafner plan. Box-Benken's plan requires 25 experiments for the same case. Maximal dimension of the vector of independent parameters is up to 20 variables.

AxPLAN™ can be used for complex analysis and optimization of multistage axial turbine flow paths operated under varying loads.

Independent variables include both geometrical and operational parameters; dependent variables or state functions are computed with different models, e.g. weight, efficiency, stresses, eigen frequencies, etc.

At the first phase, numerical experiment is planed with regard to salient features of the task. Then, complex verifying computations are carried out for different regimes of turbine operation. The computational results are transferred back to ***AxPLAN™*** for evaluation of macromodels' adequacy, interactive analyses and different kinds of optimization. Parameters variation ranges and their interaction can be re-defined, and the described process is repeated.

The procedure of optimization with DoE technique becomes most effective when DoE engine is integrated into design tool. Integrated database provides a designer with flexibility to select sets of dependent and independent variables, run computations at designer's request, immediately process and visualize the results computed, formulate and solve optimization tasks with macromodels, store, load and reuse data for repeated analyses of developed macromodels.

EXAMPLE OF DESIGN

As an illustration of these optimization and design principles, let's follow basic phases of design of 12MW axial gas turbine flow path with ***AxSTREAM™*** software suite.

In this case, the flow path design is performed automatically using such parameters as pressure and temperature at inlet, mass flow rate, speed of rotation, capacity, diameter, etc. as given.

Procedure of preliminary design provides a designer with the draft of 3-stage flow path presented in Figure 5.

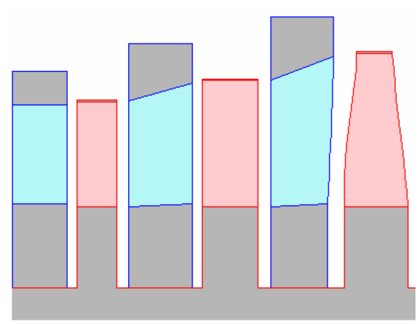


Figure 5. Draft of gas turbine flow path designed with AxSTREAM™

Let's consider more closely design of the last stage that differs by relatively long buckets and substantial flow path expansion. Figure 6 shows profiles in the stage meanline section. Dimensions of chords and blading ratio are obtained by optimization with regard to strength and vibration constraints.

Axi-symmetric analysis was carried out on 10 streamlines for different β_2 at mean radius and blade twist laws defined as parametric dependencies [$r^m ctg\beta=const$] and constant angles of nozzles (δ_1) and blades (δ_2) lean along height.

Computations assume fitting α_1 angle at mean radius to provide indicated flow rate with assigned parameters at the stage entry and exhaust.

Blade entry camber angle is assumed to be equal to flow incidence angle. The original design of the stage has no lean, and blade twist parameters were taken as $m_1=0,\ m_2=-1$ (i.e. twisting according to the law of constant vorticity).

With ***AxPLAN™*** and ***AxSTREAM™*** solvers, we built 5-parametric quadratic macromodel of gas turbine last stage that closely fit the original model data. At that, maximum of intrinsic efficiency was taken as the criterion for the blade twist optimization.

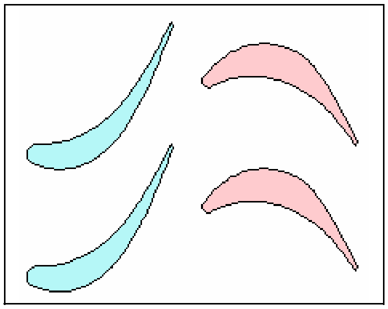


Figure 6. Profiles at mean radius of the last stage

Detail description of twist laws optimization procedure is presented in Appendix 2.

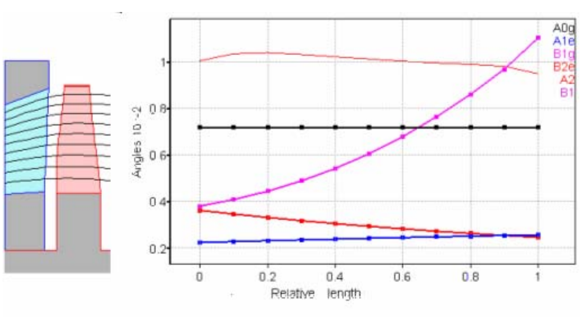


Figure 7. Streamlines and angles of the optimized last stage

Optimization with ***AxSTREAM™*** was conducted mainly in automated mode except for marginal changes of near-hub and peripheral cross-sections correction fulfilled with procedures of optimal design described above, Figure 8.

Optimal stage efficiency obtained with FMM was 83.80%, stagnation efficiency, i.e. efficiency calculated on the basis of stagnation parameters at the stage exit was 96.20%.

Computations for the same point with original model provided 83.74% and 96.17%, respectively. Comparison with original stage design demonstrates that optimization on static parameters made possible efficiency increase from 82.0% to 82.7%. An optimum is inside the selected range of independent variables variation. The optimized design features reduced reaction degree and close to axial flow exit angle.

The results of profiling shown in Figure 9 demonstrate minimal aerodynamic losses and meet stress and vibration constraints. As a reference, the data extracted from analysis with ANSYS on finite element model built with the help of ***MinuteMesh-Turbo™*** is presented in Table 1.

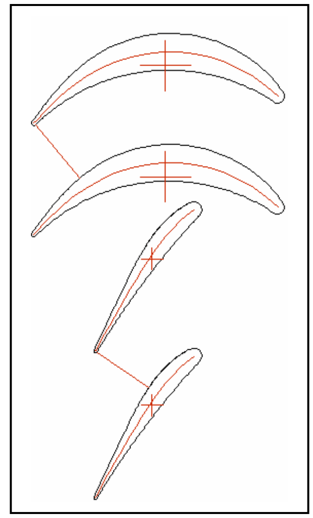


Figure 8. Extreme sections of LSB

Maximum stresses computed with 1D model are $2.5{\cdot}10^8$ Pa and almost constant along blade height. Von Misses stresses are shown by contour plot in Figure 9.

Table 1. Blade structural characteristics according to reduced order (*AxSTREAM™)* and detailed 3D (ANSYS) models.

Parameter / Solver	Max tensile stress (Pa)	Von Mises Stress (Pa)	Basic frequency (Hz)
AxSTREAM™	$2.5{\cdot}10^8$	$2.5{\cdot}10^8$	368
ANSYS	$14.8{\cdot}10^8$	$2.5{\cdot}10^8$	426

Zones where stress state is essentially three-dimensional (stress concentration at fillets) are located near blade rim and band. That is why there is a big divergence between 1D and 3D results.

As it is shown in the Table 1, Von Mises stresses defined with 1D and 3D models are identical.

Comparison of single blade basic frequency as function of centrifugal forces shows near 15% discrepancy between reduced order and 3D analyses.

Significant difference in maximum tensile stress is caused by stress concentration at the joint point of blade tip and shroud that is taken in account in 3D model and ignored in reduced order model.

As a whole, a sufficient agreement of data obtained between 3D and reduced order models warrants usage of the latter in MDO of the axial turbine flow path.

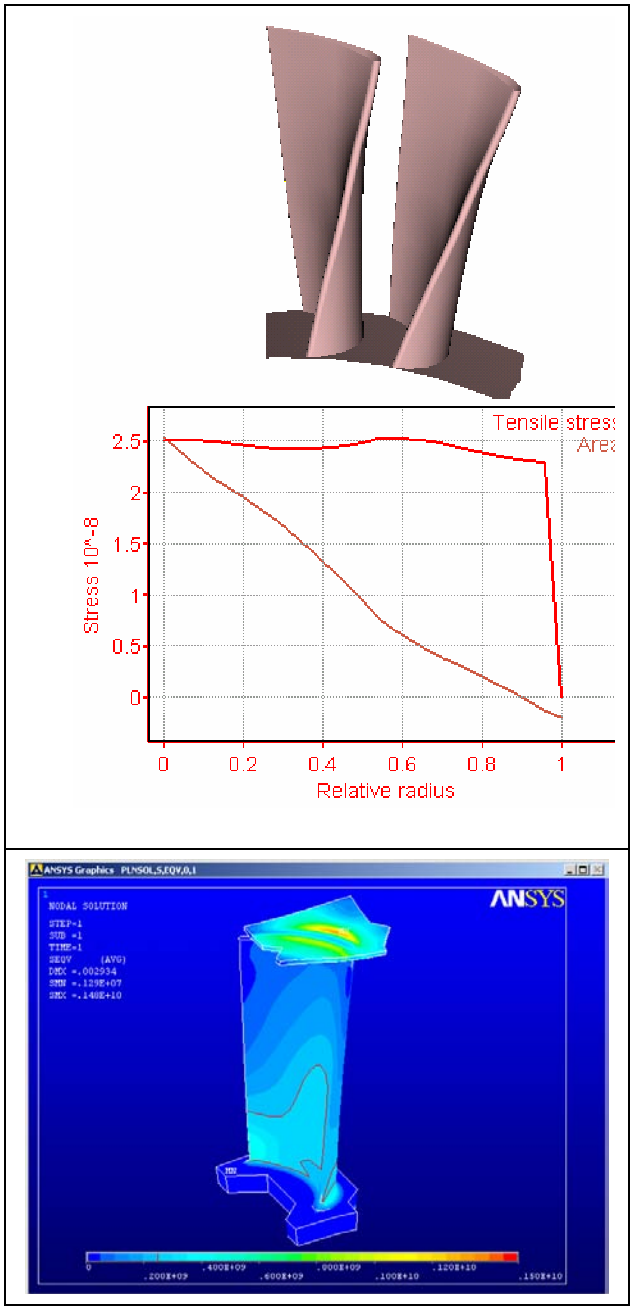


Figure 9. Distributions of last stage buckets profile sections area and tensile stress (Above - with *AxSTREAM*™, below – with ANSYS)

CONCLUSIONS

Experience gained by turbine flow path design with designer-oriented software tools described in this paper demonstrates benefits of applying reduced-order modeling approach for the flow path parameters optimization. This approach has been successfully used for anew designed turbines and modernized ones. Techniques used for optimal solution searching (DoE, random search in combination with interactive interface) can be applied to 3D optimization process. In order to address the turbine blades MDO problems with commercial CFD and FEA packages, specialized 3D mesh generators have been developed. Also specialized pre- and postprocessors facilitating parameterization and blade data transfer to a mesher and from a solver to the optimization module have been reported.

REFERENCES

1. A.V. Boiko, Yu.N. Govorushchenko. “The Theory of Axial Turbines Flow Path Optimal Design Basics.” – Kharkov, Vischa Shkola, KhSU 1989. - 217 p. (in Russian)

2. B. H. Dennis, Z.-X. Han, I. N. Egorov, G. S. Dulikravich, C. Poloni, Multi-objective optimization of turbomachinery cascades for minimum loss, maximum loading, and maximum gap-to-chord ratio, AIAA Multidisciplinary Analysis and Optimization Conference and Exhibit, Long Beach, CA, September 5-8 (2000).

3. Yiu, K. F. C. and Zangeneh, M., "A 3D Automatic Optimization Method for Turbomachinery Blade Design," *Journal of Propulsion and Power*, Vol 16, No. 6, 2000, pp. 1174-1181.

4. Janus, J. M. and Newman III, J. C., "Aerodynamic and Thermal Design Optimization for Turbine Airfoils," AIAA 2000-0840, January 2000.

5. R. Dornberger, Peter Stoll and Dirk Büche, Multidisciplinary Turbomachinery Blade Design Optimization, 38th AIAA Aerospace Science Meeting, Reno, 2000.

6. Shahpar, S., 2001, “Three-dimensional Design and Optimisation of Turbomachinery Blades using the Navier-Stokes Equations”, ISABE 2001-1053, *proceedings of 15th ISABE conference*, Bangalore, India.

7. Oyama, A, Liou, M.-S., and Obayashi, S., Transonic Axial-Flow Blade Shape Optimization Using Evolutionary Algorithm and Three-Dimensional Navier-Stokes Solver,” AIAA Paper 2002-5642, 2002.

8. A.V. Boiko, Yu.N. Govorushchenko. “The Problems of Axial Turbine Stages Design” Proceedings of USSR Academy of Science, No. 3, 1985, pp. 134-140. (in Russian)

9. A.V. Boiko, Yu.N.Govorushchenko, S.V. Yershov, A.V. Rusanov and S.D. Severin. “Aerodynamic Computation and Optimal Projection of Turbomachine Flow Paths.” - Kharkov, NTU “KhPI” 2002. - 356 p. (in Russian)

10. I.M. Sobol, R.B. Statnikov. “Optimal Parameters Determination in Multi-Criteria Problems”. Moscow, Nauka, 1981, pp. 110. (in Russian)

11. Craig H. R. M., Cox H. J. A. Performance Estimation of Axial Flow Turbines. The Inst. Of Mech. Eng/ Proc, 1970/71, vol. 185, 32/71, P. 407-424.

Appendix 1

Validation of 1D and 2D modeling accuracy

Accuracy of 1D and 2D procedures for flow path calculations is defined by the confidence level of empiric models applied for determination of cascades friction losses, leakages, etc. The loss calculation methods were selected after comparison of wind channel test results for plain cascades, field tests of stages and modules and numerical modeling results. In particular, we came to the conclusion that in most cases Craig & Cox method [11] provides quite reasonable accuracy of cascades efficiency estimation.

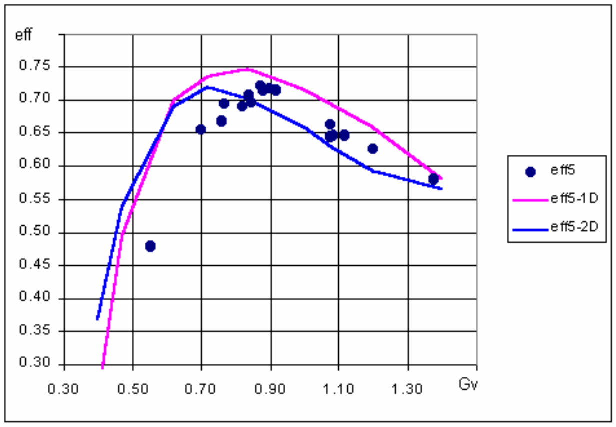


Figure10. Large steam turbine last (5th) stage efficiency calculated with 1D and axi-symmetric models at different steam volume flow rate

Our experience of computational results validation against experimental data and subsequent modification of the modeling methods assures trustworthiness of the ***AxSTREAM*** for the flow path analysis and optimization including off-design operation (Fig.10).

Appendix 2

Procedure for blade twist optimization for a stage with *AxSTREAM*

At the first glance, the optimization procedures integrated in flow path design process may seem too complex and beyond common end user comprehension. In practice, ***AxSTREAM*** operates with conventional turbine designer-oriented terminology and "walks" designer through all phases of computations that in particular include the following:

- selection of 1D or axi-symmetric problem formulation , Fig. 11;
- assignment of independent variables and response functions for building the formal models with the help of DoE methodology, Fig. 12;
- selection of parameters variation ranges for creation of quadratic models and estimation the error of approximation, Fig. 13 and 14;
- optimization problems formulation and solution with formal models and subsequent results verification with baseline model, Fig. 15.

In the example discussed here, such blade twist parameters (m_1 , β_{2mid}, m_2 and blade lean angles) should be found that could provide maximum efficiency at constrained hub reactivity, Fig. 15.

The results of computation with original model, Fig. 17, and formal, Fig. 16, models at the optimal point have a good convergence.

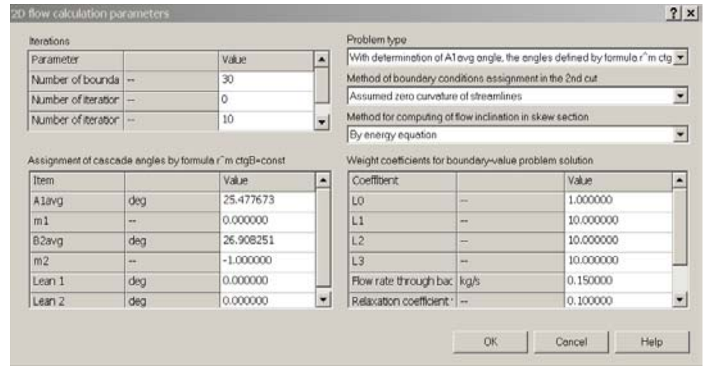


Figure 11. Selecting problem formulation for stage axi-symmetric analysis

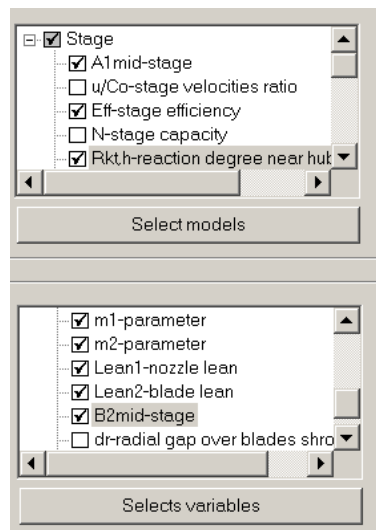


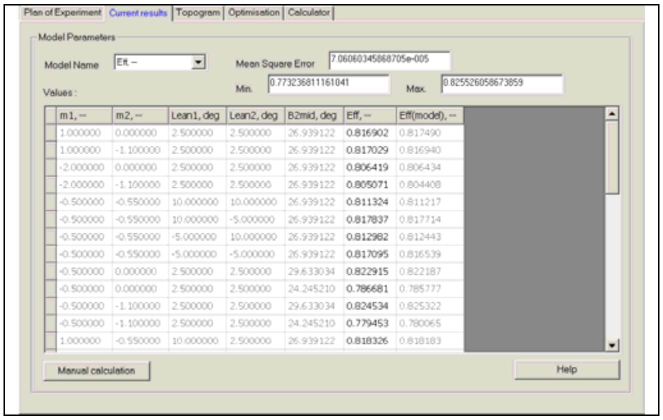
Figure 12. Assignment of the type of model and independent variables

Var. Name		Min	Max
m1	--	-2.000000	1.000000
m2	--	-1.100000	0.000000
Lean1	deg	-5.000000	10.000000
Lean2	deg	-5.000000	10.000000
B2mid	deg	24.245210	29.633034

Figure 13. Selecting ranges of independent variables variation

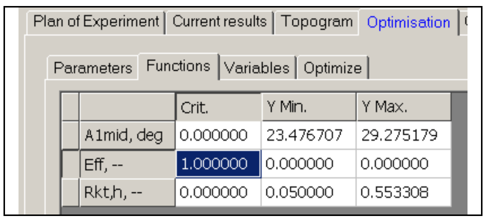
In conclusion, a geometric interpretation of the optimal solution on the plain of blade twist parameters m_1 и m_2 is presented, Fig. 18. It demonstrates how sensitive the efficiency to blade twist variation is, and how seriously a constraint on hub reaction degree impacts the optimal solution.

The example discussed here were run on commercial AMD Athlon 2100+ processor. Axi-symmetric computations for response functions extraction took less than a minute (41 points for 5 independent variables).



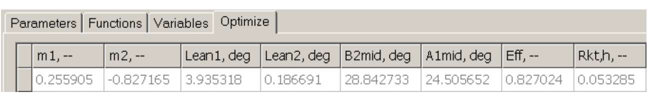
m1, --	m2, --	Lean1, deg	Lean2, deg	B2mid, deg	Eff, --	Eff(model), --
1.000000	0.000000	2.500000	2.500000	26.939122	0.816902	0.817490
1.000000	-1.100000	2.500000	2.500000	26.939122	0.817029	0.816940
-2.000000	0.000000	2.500000	2.500000	26.939122	0.806419	0.806434
-2.000000	-1.100000	2.500000	2.500000	26.939122	0.805071	0.804408
-0.500000	-0.550000	10.000000	10.000000	26.939122	0.811324	0.811217
-0.500000	-0.550000	10.000000	-5.000000	26.939122	0.817837	0.817714
-0.500000	-0.550000	-5.000000	10.000000	26.939122	0.812982	0.812443
-0.500000	-0.550000	-5.000000	-5.000000	26.939122	0.817095	0.816539
-0.500000	0.000000	2.500000	2.500000	29.633034	0.822915	0.822187
-0.500000	0.000000	2.500000	2.500000	24.245210	0.786681	0.785777
-0.500000	-1.100000	2.500000	2.500000	29.633034	0.824534	0.825322
-0.500000	-1.100000	2.500000	2.500000	24.245210	0.779453	0.780065
1.000000	-0.550000	10.000000	2.500000	26.939122	0.818326	0.818183

Figure 14. Computational data for response functions generation



	Crit.	Y Min.	Y Max.
A1mid, deg	0.000000	23.476707	29.275179
Eff, --	1.000000	0.000000	0.000000
Rkt,h, --	0.000000	0.050000	0.553308

Figure 15. Selecting criteria and functional constraints for optimization problem solution



m1, --	m2, --	Lean1, deg	Lean2, deg	B2mid, deg	A1mid, deg	Eff, --	Rkt,h, --
0.255905	-0.827165	3.935318	0.186691	28.842733	24.505652	0.827024	0.053285

Figure 16. Results of optimization with quadratic models

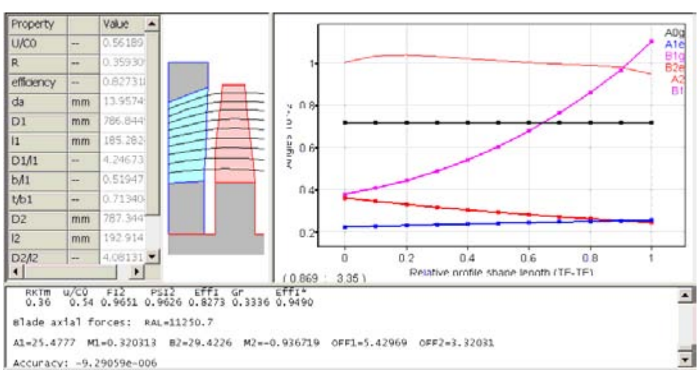


Figure 17. Results of computation at optimal point with original model

A random search with quadratic models was performed within 1 second (100,000 points). Hence, time that a designer typically spends on design and optimization process without automated tools can be used for addressing such creative tasks as optimization problem formulation and final results analysis and interpretation.

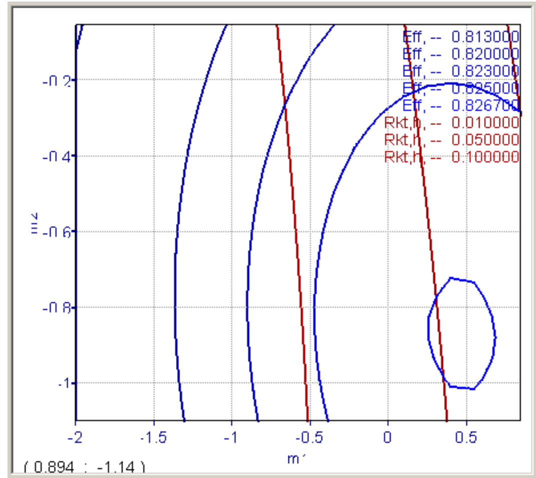


Figure 18. Geometric interpretation of optimal solution in the plain of m_1 and m_2 parameters at constrained hub reactivity